

Antiquity

John O'Reilly

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Song Collection

Dedicated to:

the greats of
mathematics, science and philosophy
and the many associated historians.

john.oreilly@majorsoftware.com.au

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Prelude

a wealth of maths history
intertwined with science & philosophy
introduced to me in my
teacher-college years,

in search of clarity, secured
a screed of history of maths texts
and a need to write.

hand written, (so few *pc's back then*
'Pythagoras and Beyond'. utilised
as extensions for my students

inspired 'Taking on the Challenge, Parts 1-3'.
looking to gap the logic
between various starting premises
and methodologies potentially used

some content published in
‘TENRAG – A Mathematics Journal
for Students’, with my thanks to:
Garnet J. Greenbury.

then looked to link this search for clarity
with a love of music.

14 songs, in search of an indi-rock band,
presented chronologically –
starting with Thales and Pythagoras,
closing out with Hypatia.

ever aware that these songs portray
a history, the lives of significant people,
cultures and times
that i was not directly privy to
please accept these as
partial interpretations,
not in any way complete or definitive.

hoping to inspire someone somewhere
to truly appreciate the
texture and vibrancy
that this history brings to the
discipline and rigour of
maths and science.

an enormous amount of energy was
expended by a millennium of
deep thinkers...

to be honoured and celebrated.

for me – it is something to sing about...

Pythagoras

Ionian influence, intellect and art,
Pupil of Thales, structure and heart,
Babylonian inspiration, take Egypt too,
A touch of India, the vision drew.

Standing tall in the place of the brotherhood,
Mystic of maths and philosophy,
Carving out an order of secrecy,
Drawing the local rich nobility.

Divine the number, perfect some prime,
2 and 3 for gender, union makes 5,
Sum the factors, 1, 2 and 3,
Shape the sequence, geometric formulae.

Taking squares on the sides of triangles,
Challenging us to prove it true,
They then put their faith in geometry,
On chasing the square root of two...

Like maths like music, logic and art,
Harmonic Proportion, $\frac{2}{3}$ and $\frac{1}{2}$,
On string a fifth, or octave enquired,
Musical Science, Egyptian inspired.

Senses deceiving, illusive and fake,
Not the trader, nor competitor break,
Only in logic, with thought and with sleuth,
Can philosophers find, an absolute truth.

Drawing squares on the sides of
triangles,
Beginnings now bathed in mystery,
A society, a theorem, a philosophy,
A secured place in history.

Parmenides and Zeno

So is time infinitely divisible?
Or is it made up of atomic instants?
Zeno asked – is it possible to travel through
an infinite number of divisions in a
finite time?
If the tortoise is one tenth as fast as the
hare, can the tortoise ever pass?

Taking on Pythagorean philosophy,
Parmenides challenges from Southern
Italy,
Arguing against the world's state of flux,
More fixed, unchanging was the crux.

So is motion also infinitely divisible?
Or is it made up of atomic frames?
Zeno asked – do we walk about in space one
frame at a time?
Can we be sure about these frames once we
allow in some relativity?

Zeno offered a paradox or four,
Put to question what was claimed
before,
Built up a view so logical,
Tested the axioms from the resulting
consistency.

So... what is truth?
What is truth?

Heroic Age

We have a problem, please come and see,
To square a circle, it cannot be,
Anaxagoras, you set the pace,
But ruler and compass, they've lost their
place.

We have a problem, please come & sanctify,
To duplicate this cube, each side to modify,
To avert this plague, you must devise,
An altar for Apollo, double its size.

We have a problem, please try to free,
Trisect this angle, cut this in three,
Ruler and compass, Hippias, not them alone,
With a trisectrix, your method is known.

We had a team, and they took the field,
So little to start, yet so much revealed,
We had these problems and you came and
stayed,
Not all were solved then, but the paths were
laid.

Socrates

I might believe you, I might deceive you,
I can lead you either way;
May be a sophist, may be a prophet,
There's no truth here to be found.

Let me challenge you on what you hold,
Let me scrutinise what you've been told,
Lest you realise the futility,
There's no place here for your certainty,
Just humanity.

I challenged poets, I challenged artists,
I took to politicians,
It's about delivery, it's about philosophy,
It's all about what god demands of me.

Let me tell you while I honour you,
People of Athens, I must be true,
There is knowledge and it's divine,
It's just not that it's yours or mine,
We're just humanity,
We're just humanity.

Plato

Across the door it said:
'Let no one ignorant of
geometry enter here'.

In Utopia bronze labourers,
silver soldiers gold guardians,
with inheritance and gender mix.

And as you drew on those before,
You brought structure and so much
more,
Yours are the shoulders on which we
stand,
To view our world and to understand.

Logic & religion. Concepts & terms,
Deceptive senses leave us devoid,
Eternal truths through love of wisdom.

Immortality morals, ethics, religion,
On suicide on body & soul,
On transmigration and the creator's role.

And they took you at your word,
Or they took on what then served,
Let your thoughts have their day,
In doctrine so many fell to prey.

Cosmology four elements,
five regular solids one earth one fire,
one water one air one the universe.

And you gave them a place to be,
To think through maths and
 philosophy,
To chase their proofs in geometry,
To deliberate truth through eternity.

Eudoxus

From the streets of Cnidus,
Came a humble yet able lad,
Keen to bend some thinking,
And give of what he had.

With ratio and proportion,
He helped deal with the square root of two,
Extended for irrational numbers,
Gave Euclid a way on through.

Add value to what is there, and pass it
through,
Open up your mind, and see what's true,
See what is true...?

With polygons and circles,
Came a lemma that paved the way,
For Euclid and Archimedes,
And others of modern day.

With a start on integration,
On diameters, circles and spheres,
Cones and cylindrical volumes,
A great, gave of his years.

Rising to the challenge, and face the
fears,
Daring to chase a calling, and persevere
And to persevere.

And then Plato posed a problem,
To consider a view of space,
With the earth at its centre,
Circular motion & uniform pace.

To map centuries of data,
Again Eudoxus paved a way,
This father of scientific astronomy,
For those of more recent day.

Take in what's handed down, and see
what's grey,
Build up a paradigm, or try another
way, Pose another way...

Menaechmus and Dinostratus

From the teaching of Eudoxus,
Menaechmus and Dinostratus,
Solved the Duplication of the Cube,
And the Squaring of the Circle.

They made use of the tools available,
Looked to solve what was achievable,
Focussed first on what was workable,
Kept an eye on the deliverable.

From the schools of Plato and Aristotle,
Came so many who played a part,
In advancing maths and philosophy,
And science, in particular astronomy.

Aristotle

You were grown from some stellar minds,
and then some,
He dubbed you: 'Intellect of the School',
for your brilliance,
You revised his mysticism,
and theology,
You added original points of view,
and wrote them out.

Universals were a ‘sort’ of thing,
like cats and dogs,
Essence was the characteristics,
the given nature,
Substance was the very ‘thing’,
like a cat or dog.

They passed you down through time,
Paying homage to your every line,
Favourable critique for the rhyme,
So divine.

Form gave unity to a set of matter,
gave it essence,
Bare matter had potentiality,
constant evolution,
The universe was forever evolving,
to a greater form,
God was pure form and actuality,
and could not change.

Soul was the form of the body,
with a single purpose,
The mind was placed higher again,
could grasp what's true,
Leading you to a view of motion,
and the existence of God.

Something must originate motion,
 Something like an 'Unmoved Mover',
 A pure form as the final 'Cause',
 Look to God.

‘Good’ was aligned with happiness,
an activity of soul,
‘Virtue’ was being intellectual,
leads to teaching,
‘Virtue’ was also being moral,
resulting from habit,
‘Virtues’ were a golden mean,
between two extremes.

Justice was a sense of right proportion,
for given stations,
A list of traits for those superior,
such as a sense of pride,
Refrain from gossip and a need for praise,
and asserting blame.

‘Good’ was obtainable by the few,
(the province of)
For any who would rise to this level,
Not tied or defined by heritage,
Just those superior.

State is the greatest form of community,
encourage noble action,
Allow each member to fulfil their purpose,
for the greater good,
To endorse a respect for the law,
and proper Justice,
Those destined to be citizens of the state,
mould through learning.

You added syllogism to the art of logic,
a 3 part argument,
Say all songs are for singing,
and this is a song,
We conclude then that for this song,
this song's for singing.

The start of our formal logic,
 Good start but not complete,
 Other forms were then to follow,
 More to follow.

Your physics was a focus on nature,
potential and movement,
A seed is the potential of a living plant,
its growth its movement,
Fulfilling what exist potentially by fate,
is nature's motion,
Originating from an internal principle,
arriving at some completion.

In maths you worked ‘on indivisible lines’
and mechanical problems,
The separation of arithmetic and geometry,
and systems of logic,
On a proof around the $\sqrt{2}$...
could not be fractioned.

Providing a basis on which to start,
The reviewers, they played their part,
Taking in, working through, taking part
Then building on...

Euclid

Draw a straight line,
Point to point fine,
Set down a postulate,
Angles on intersects,
History then introspects,
Set down a solid base.

Euclid Euclid.

Equal to the same thing,
Must be the same thing,
Call this a common notion,
Add equals to equals,
Wholes must be equal,
Some five were specified.

Euclid Euclid.

Chorus

Elements to build on,
 Elements that grew from,
 Centuries of intellect,
Theorems built headlong,
 Geometry so strong,
 Withstanding our tests of time,
While true in its own right,
 It pathed out a new light,
 For others with the gall to try,
Truth is the real curse,
 Say what of the universe,
 Draw out our space and time.

Take the *Elements* off the shelf,
 Build a Paradigm.

Building on axioms,
 Assumptions to overcome,
 Logic plays its part,
Proofs to accumulate,
 Books to populate,
 Thirteen is a solid start.

Euclid Euclid.

Proof of Pythagoras,
 On circles was rigorous,
 Geometric algebra,
Construction in circles,
 Then incommensurables,
 Polygons and cylinders.

Euclid Euclid.

So question the axioms,
Look for a premium,
The truth beyond the veil,
Completeness & consistency,
Independence to guarantee,
Deception cannot prevail.

Euclid Euclid.

Saccheri stood up first,
Others then became immersed,
Non-Euclidean was the call,
Warping our time and space,
Alternate views to embrace,
So strange but logical.

Euclid Euclid.

Archimedes

Find him a place to stand,
and he'll move the earth;
Give him a crown of gold,
and he'll find its worth.

Threaten them a Roman Fleet,
and he'll take it on;
Leveraging mechanical might,
will see them gone.

Chorus 1

You have changed the world on your own
terms, given of...

You have borne your truths, trial by
time confirms, stated proof;

Maths applied and pure, so many
theorems, delivered on.

First of three, Historically,
A title to behold.

Suggest him anything a 'weapon',
and it'll help him solve;
Want water up the banks of the Nile,
mechanics will evolve.

Give him a ship fully laden,
with a need to move;
Give him the head of the pulley,
he'll draw straight and smooth.

Chorus 2

Plutarch reported your story in some
detail, life and times;
Marcellus forced out your mechanical
genius, weapons of war;
Catapults and mirrors and clawed
devices, brought to bear.

First of three, Historically,
A title to behold.

Give him the range of mathematics,
and he'll think it through;
On circles, spheres and cylinders,
parabola, just to name a few.

On the equilibrium of plane figures,
fulcrums and levers;
On the famous floating bodies,
The Method and Book of Lemmas.

Chorus 3

You gave us so many methods, so many
treatises, deliberated on;
Method for calculating pi, constructing a
heptagon, with its proof;
New methods around so many older
problems, applied and pure.

First of three, Historically,
A title to behold.

Apollonius of Perga

Quick delivery,
Cutting off of a Ratio,
Cutting off of an Area,
Determinate Section,
Tangencies, Vergings,
Plane Loci,
and Conics.

The Great Geometer.

Conics brought to bear:
Parabola, hyperbola,
Ellipse, all cut from the same cone,
Plus a definition for the double napped cone,
in Conics.

Apollonius of Perga.

Book III presented theorems of solid loci,
Determinations of limits,
Book V: a theorem,
On the normal to a parabola,
in Conics.

The Great Geometer.

At the end of the Golden Age,
At the end of the Golden Age,
At the end of the Golden Age,

The Great Geometer.

Alexandrian Astronomy

Geocentric Models

Earth holds the central position of the
universe,
Planetary motion is described by perfect
circles,
Plato set it out and it became solid doctrine,
Built-on by Eudoxus and tidied by Aristotle,
Epicycle and Eccentric Motion added by
Apollonius.

Devise what we cannot see, empirically,
Model then a universe, incrementally,
Accept that the final truth hides
deceptively,
Alexandrian Astronomy.

Heliocentric Models

The sun takes on central position in the
universe,
Earth rotates once, on axis, every day,
Earth is tilted and is just one of the planets,
Aristarchus seemed to peg it, so logical,
Those of his time though, thought it
contestable.

Step away from the doctrines,
constructively,
Build up again from principles,
creatively,
Be open to a view, that's new, with
consistency,
Alexandrian Astronomy.

Size Distance Estimation

Aristarchus improved on the (*estimation of
the*) size of the sun and moon,
Eratosthenes made close the diameter of the
earth,
Went on to estimate distances to the sun
and moon,
Hipparchus climbed a mountain to estimate
the size of the earth,
Using this in turn to estimate the distance
to the moon.

Take on the impossible challenges,
decisively,
See it, measure it, think it, intelligently,
Nothing seems too large, too small,
indefinitely,
Alexandrian Astronomy.

This is not just a story of Antiquity,
Copernicus and Kepler made further
delivery,
Even now we continue to restructure our
reality,
Building up our Astronomy.

Silver Age

It took three centuries to re-engage,
Opening up to – the Silver Age;
Menelaus reportedly jumped on board,
Spherical triangles, calculations of chords;
Nicomachus's arithmetic became philosophy,
Pythagorean teaching revived purposefully.

Heron approached things very practically,
Pneumatics, wrote on physics extensively;
Drilled a tunnel both sides, simultaneously,
Worked applied and pure mathematically.

Chorus

Notwithstanding times, a turbulent space,
Legacy strong, foundations in place,
Consolidation due, challenges to face,
New deliberations, new work to embrace.

Ptolemy built foundations for trigonometry,
Gave geometric structure for astronomy;
Worked on optics, vision psychology,
Wrote an 8 book masterpiece on geography;
Diophantus known as the father of algebra,
Set foundations for later work in Arabia.

Pappus was the last great geometer,
'Mathematica Collections' prepared in
 Alexandria;
 Problems were plane, solid and linear,
 With proofs and lemmas, reflections from
 nature.

Hypatia

Closing out the Silver Age,
Was one who took to pen a page,
On such as Apollonius and Ptolemy.

Reported that time was spent,
On more than one scientific instrument,
Among which was an Astrolabe.

Chorus

And at the time you were,
With fostered talent, to transfer,
A popular Alexandrian lecturer.

With time spent on geometry,
Your father passed on his legacy,
To deliberate rigorously.

Trained you in both body and mind,
You delivered your thoughts to humankind,
In maths, in science, in things designed.

You delivered on your commentaries,
Debated contemporary philosophies,
Built on prior analyses.

You stood your ground in arguments,
Took on philosophical entanglements,
But the times, they say, were turbulent.

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